



Lecture 12 part 2 - Modelling and control of Wave Energy Converters

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Modelling and control of Wave Energy Converters

By Morten Kramer

PhD course

"Numerical and experimental modelling and control of Wave Energy Converters"

Tuesday 1 September 2015

Location: Ecole Centrale Nantes, Nantes, France

Energy production by wave energy converters

Wave climate

Hanstholm site. Wave probability [%]									
Wave height H_{m0} [m]		Wave period $T_{0.2}$ [s]							All
		2-3	3-4	4-5	5-6	6-7	7-8	8-9	
		2.5	3.5	4.5	5.5	6.5	7.5	8.5	
0.0 - 0.5	0.25	2.65	8.18	1.84	0.38	0.14	0.03	0.01	13.23
0.5 - 1.0	0.75	1.22	19.22	11.44	2.21	0.18	0.06	0.02	34.35
1.0 - 1.5	1.25	0.00	6.84	13.07	2.96	0.30	0.04	0.00	23.21
1.5 - 2.0	1.75	0.00	0.33	9.59	3.05	0.29	0.04	0.00	13.30
2.0 - 2.5	2.25	0.00	0.02	3.34	4.60	0.20	0.04	0.00	8.20
2.5 - 3.0	2.75	0.00	0.01	0.22	3.89	0.21	0.01	0.01	4.35
3.0 - 3.5	3.25	0.00	0.00	0.00	1.39	0.51	0.01	0.01	1.92
3.5 - 4.0	3.75	0.00	0.00	0.00	0.17	0.58	0.02	0.01	0.78
4.0 - 4.5	4.25	0.00	0.00	0.00	0.00	0.25	0.07	0.00	0.32
4.5 -	4.75	0.00	0.00	0.00	0.00	0.07	0.21	0.05	0.34
All		3.87	34.60	39.50	18.65	2.73	0.53	0.11	100.00



Power matrix

Wavestar C6-600 20 float, 70 % PTO, Storm protection $H_{m0} = 3.5$ m. Electrical power [kW]									
Wave height H_{m0} [m]		Wave period $T_{0.2}$ [s]							
		2-3	3-4	4-5	5-6	6-7	7-8	8-9	
		2.5	3.5	4.5	5.5	6.5	7.5	8.5	
0.0 - 0.5	0.25	0	0	11	13	13	13	13	
0.5 - 1.0	0.75	19	46	71	85	89	88	84	
1.0 - 1.5	1.25	50	121	182	212	215	206	192	
1.5 - 2.0	1.75	95	231	339	381	374	350	322	
2.0 - 2.5	2.25	154	375	535	579	554	511	466	
2.5 - 3.0	2.75	228	552	600	600	600	600	600	
3.0 - 3.5	3.25	319	600	600	600	600	600	600	
3.5 - 4.0	3.75	0	0	0	0	0	0	0	
4.0 - 4.5	4.25	0	0	0	0	0	0	0	
4.5 -	4.75	0	0	0	0	0	0	0	

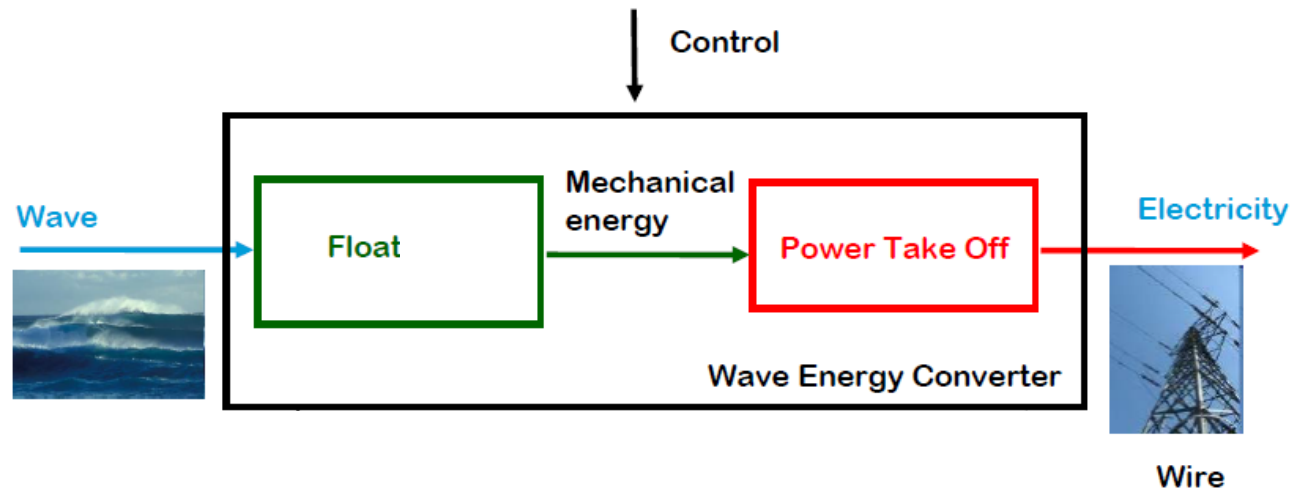
$$* \frac{24 * 365 \text{ h/year}}{10^5} \Rightarrow$$

Wavestar. Predicted energy production at Hanstholm [MWh/year]									
Wave height H_{m0} [m]		Wave period $T_{0.2}$ [s]							All
		2-3	3-4	4-5	5-6	6-7	7-8	8-9	
		2.5	3.5	4.5	5.5	6.5	7.5	8.5	
0.0 - 0.5	0.25	0	0	2	0	0	0	0	2
0.5 - 1.0	0.75	2	78	71	16	1	0	0	169
1.0 - 1.5	1.25	0	72	209	55	6	1	0	342
1.5 - 2.0	1.75	0	7	285	102	9	1	0	404
2.0 - 2.5	2.25	0	1	156	233	10	2	0	402
2.5 - 3.0	2.75	0	0	12	204	11	1	1	229
3.0 - 3.5	3.25	0	0	0	73	27	1	1	101
3.5 - 4.0	3.75	0	0	0	0	0	0	0	0
4.0 - 4.5	4.25	0	0	0	0	0	0	0	0
4.5 -	4.75	0	0	0	0	0	0	0	0
All		2	158	734	684	64	5	1	1650

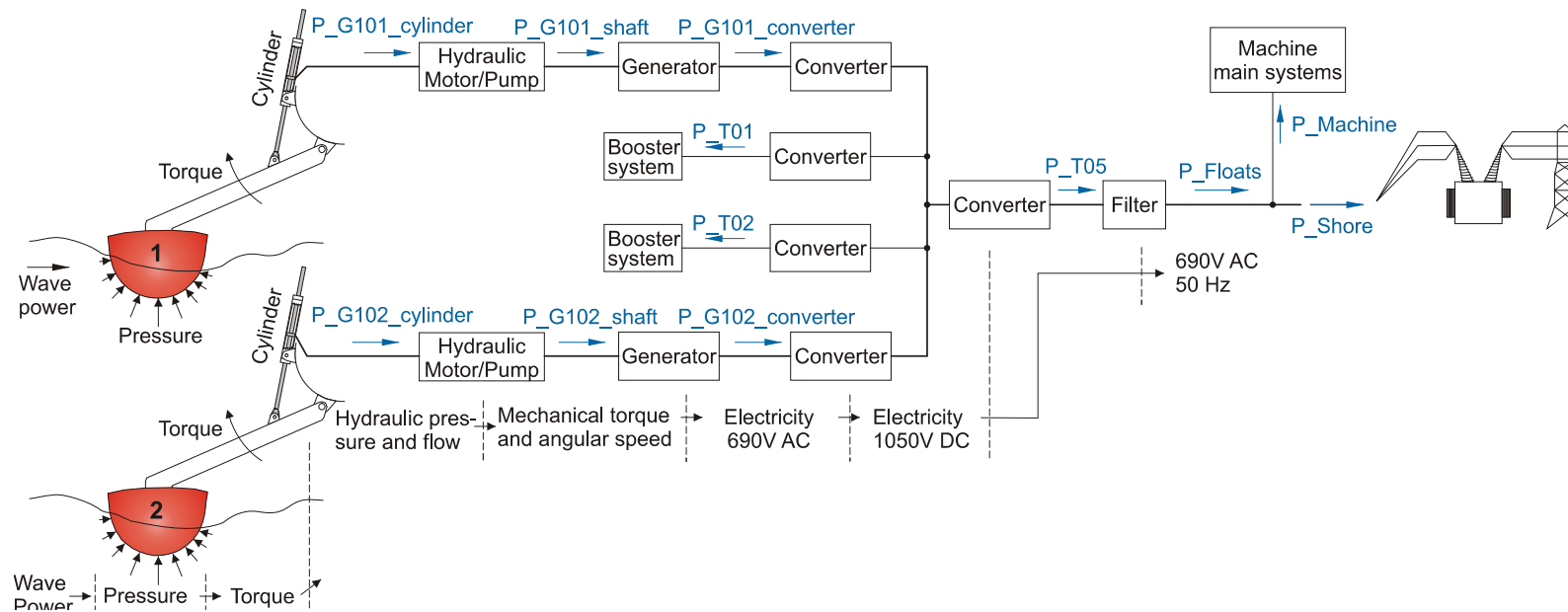
**Yearly
production: 1650
MWh**

(Excluding: Periods out of operation due to maintenance or faults, and background consumption when in idle mode)

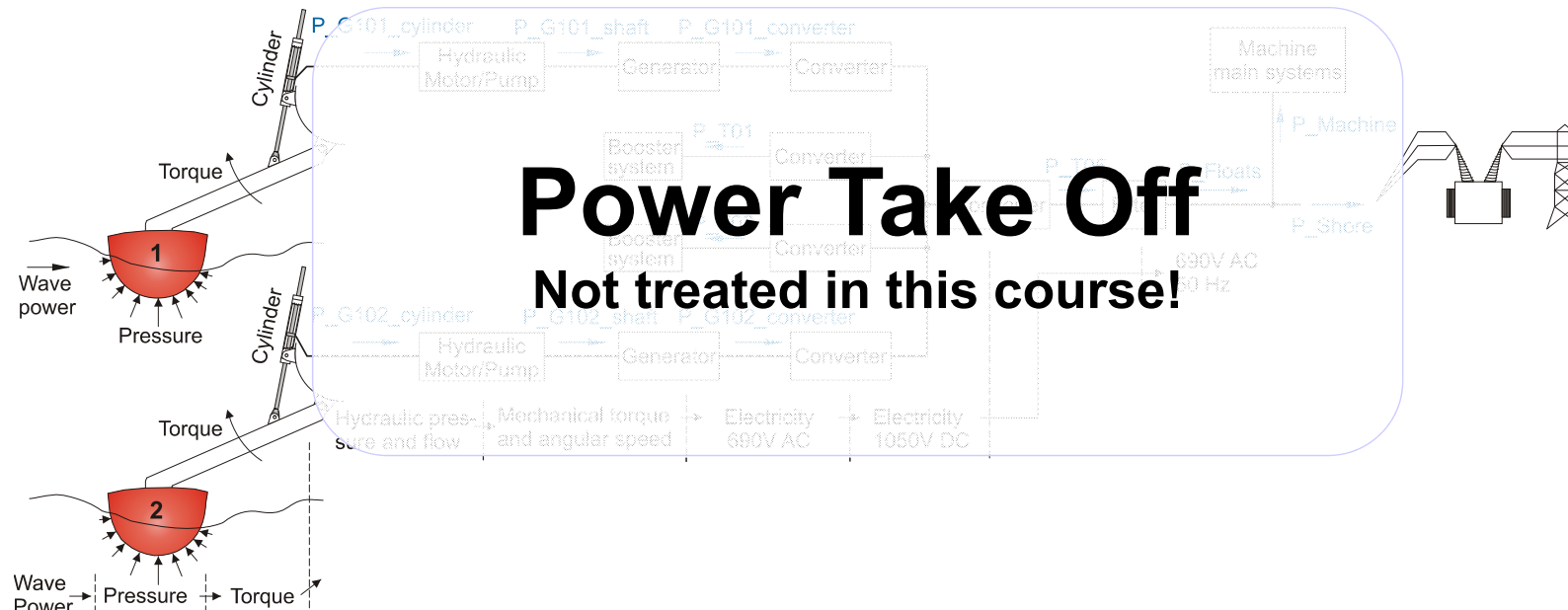
How can we model this?



Wavestar extracts the wave power and converts it to electricity



What will you learn today?

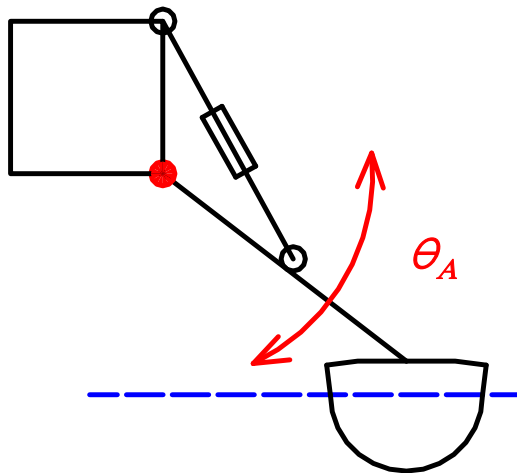


Today you will learn how to calculate the optimum force to apply on the Wavestar absorbers in order to maximize the average power production in terms of absorbed mechanical power.

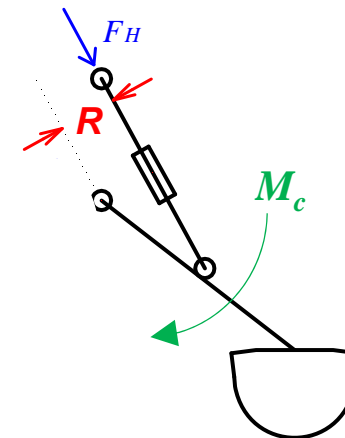
Focus is on linear damping control and reactive control for the point absorbers, but the methods apply for most types of WECs.

Kinematics of the Wavestar floats

The Wave Star floats rotates around a bearing:



The motion is controlled using the force from the hydraulic cylinder $M_c = R \cdot F_H$:



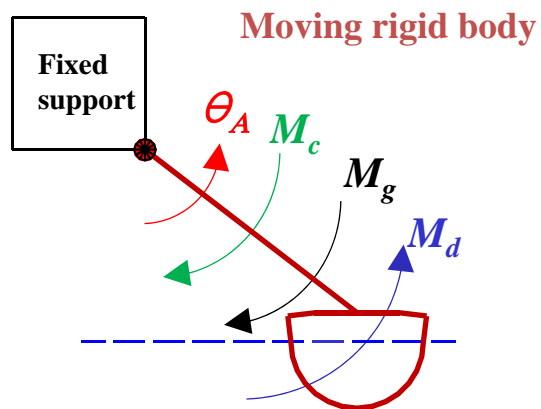
Control problem: To maximize the power transfer (by controlling M_c)

The instantaneous absorbed power is calculated by:

$$P_a(t) = M_c(t) \cdot \dot{\theta}_A(t)$$

Maximal power absorption is achieved when $M_c(t)$ and $\dot{\theta}_A(t)$ is in phase!

Mathematical model of the Wavestar floats (force balance)



Pivoting motions are described by angular rotations θ_A .

Newton's second law:

$$J\ddot{\theta}_A = M_d - M_g - M_c$$

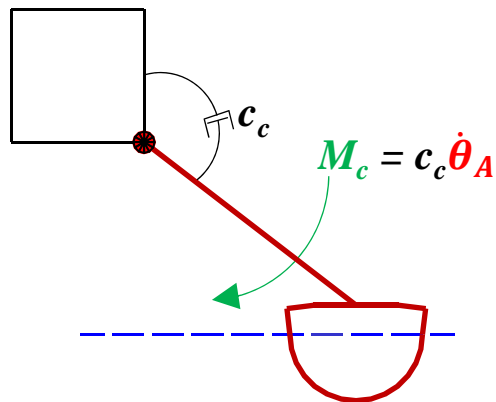
- J : Mass inertia moment of the moving body
- $\ddot{\theta}_A$: Angular acceleration
- M_d : Hydrodynamic moment (from water pressure)
- M_g : Gravitational moment
- M_c : Control moment from Power Take Off

We would like to choose the control moment M_c in a way to take out as much energy as possible.

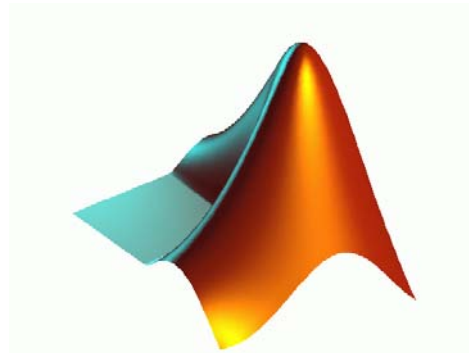
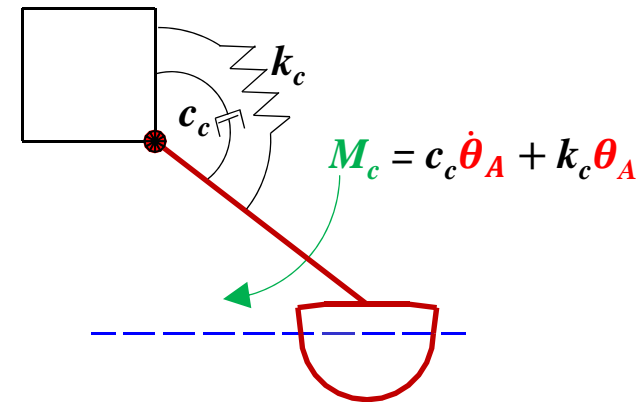
The method for choosing the moment is referred to as “control strategy”.

Two simple control strategies with constant gain factors

Resistive control
(linear damper-system)



Reactive control
(linear spring-damper-system)



Demonstration with GUI

Purpose of gains:

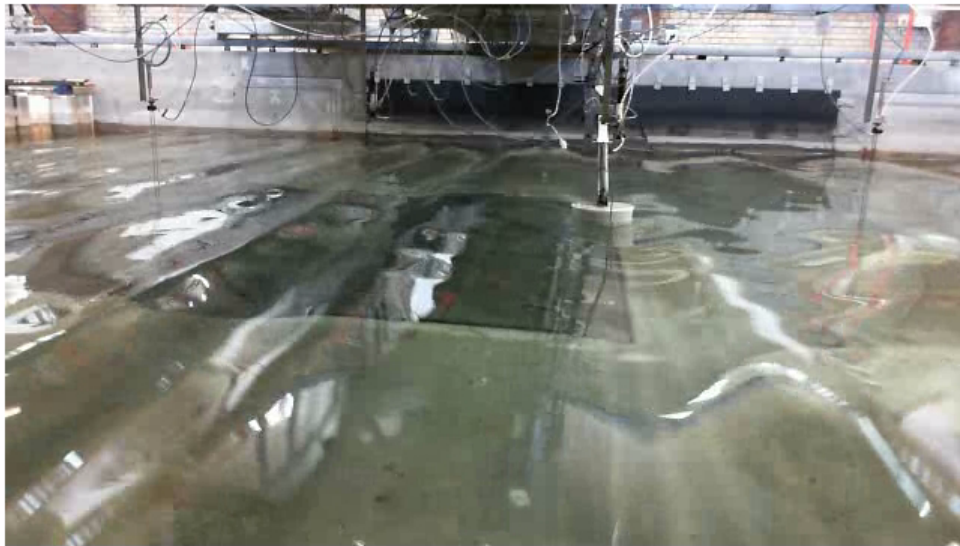
Resistive part (c_c): Power absorption

Reactive part (k_c): Change system frequency

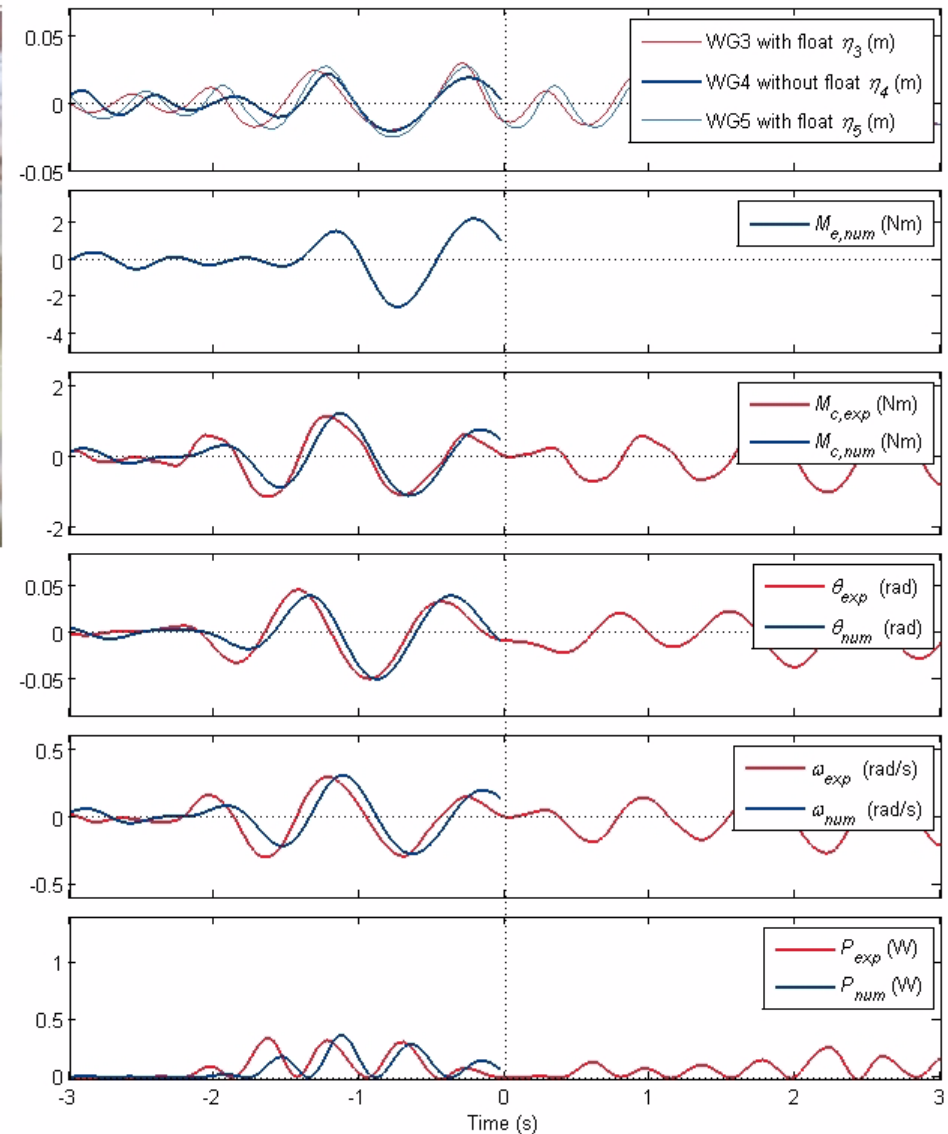
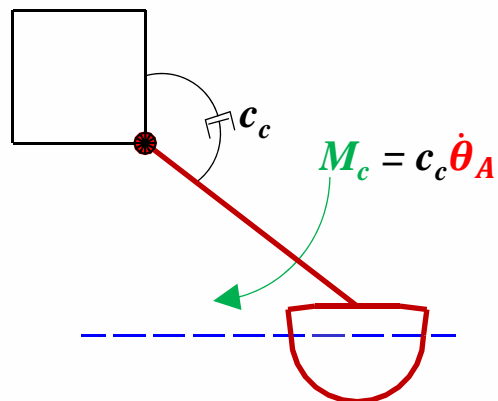
Example of laboratory tests with Wavestar floats

Linear damping control $c_c = 4.0 \text{ Nm/(rad/s)}$, wave dir 0° , IRB1 ($H_{m0} = 0.051 \text{ m}$, $T_p = 1.0 \text{ s}$).

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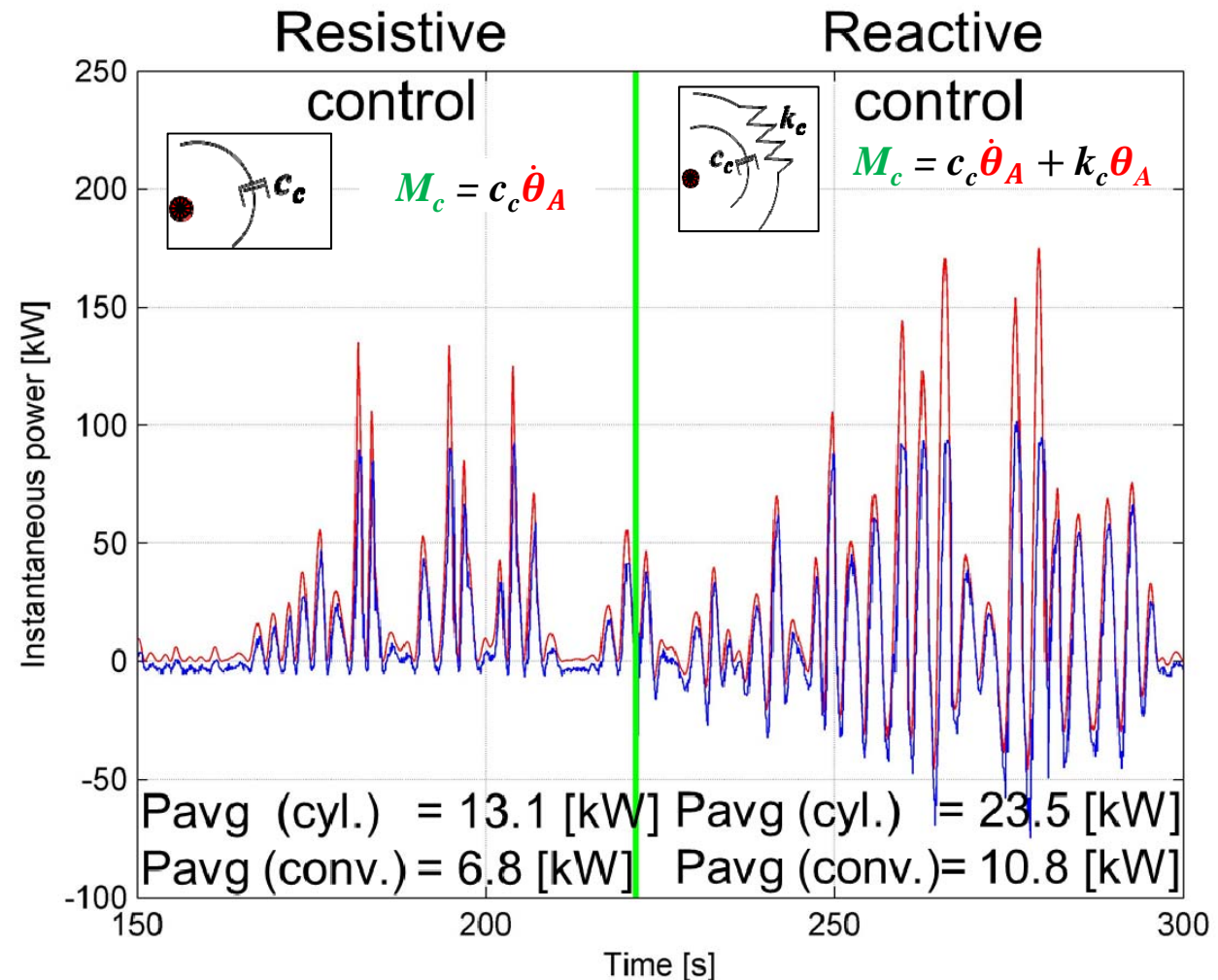


Resistive control
(linear damper-system)

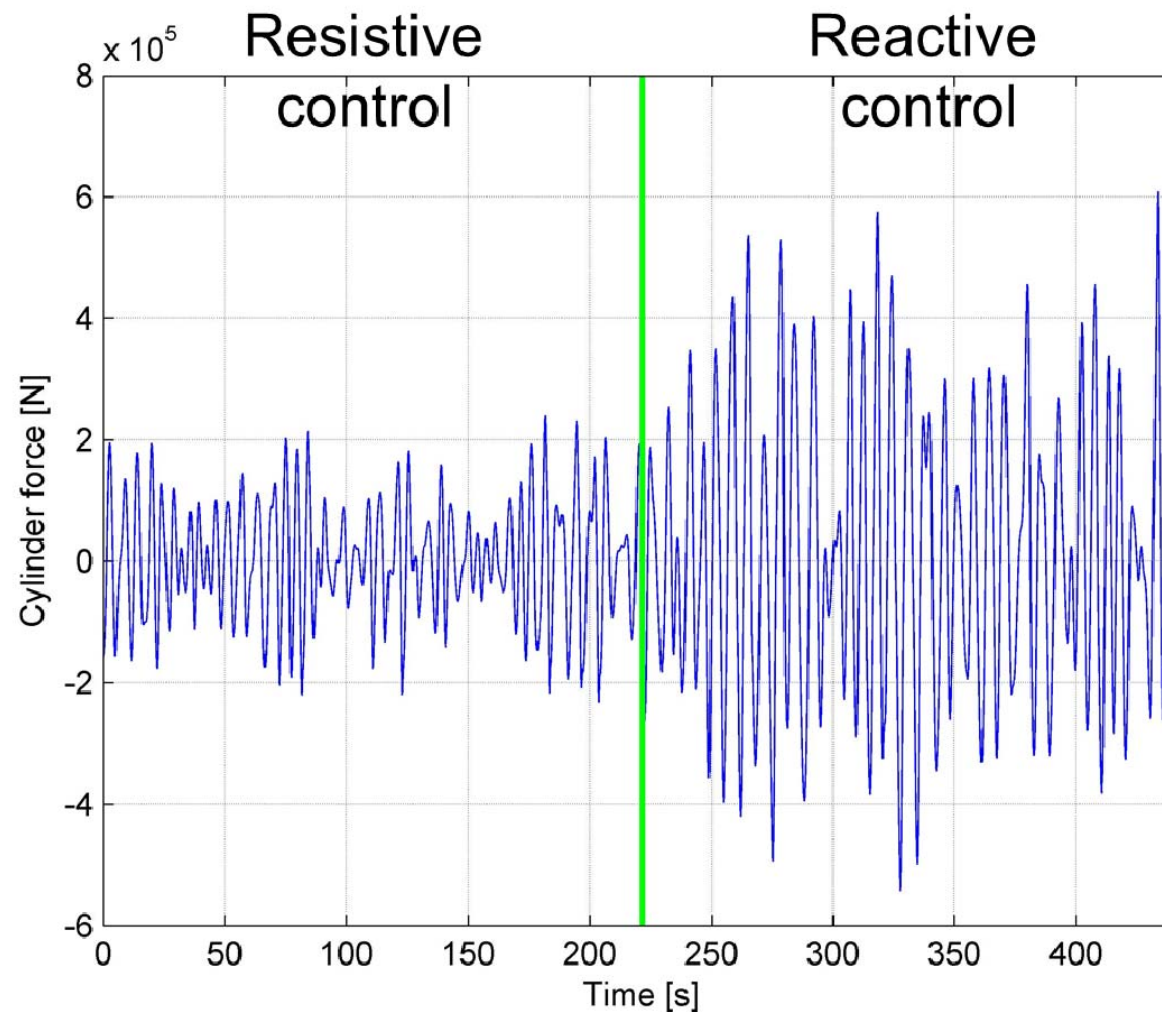


Power absorption depends strongly on the choice of control strategy

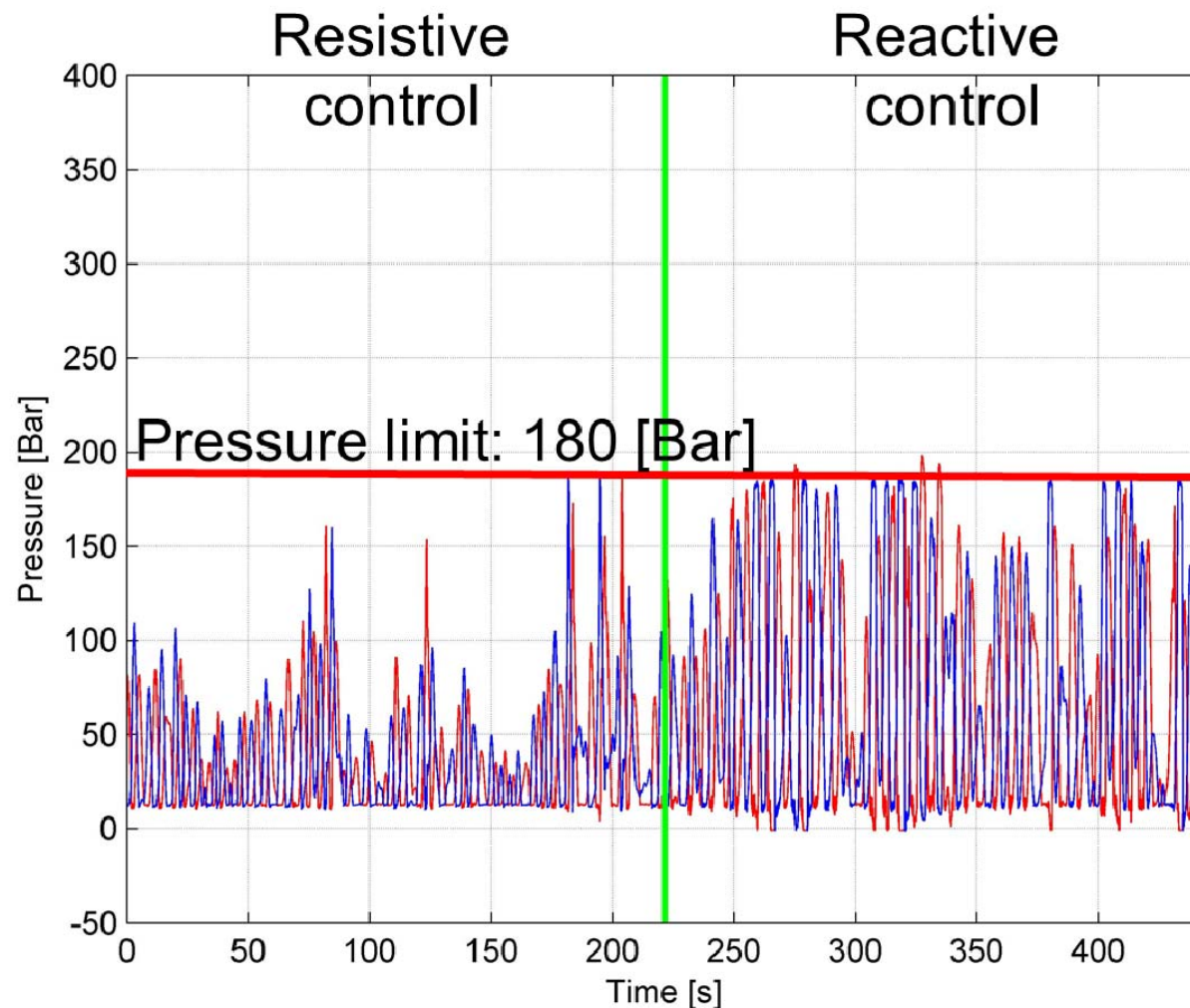
Example of measurements from Wavestar at Hanstholm



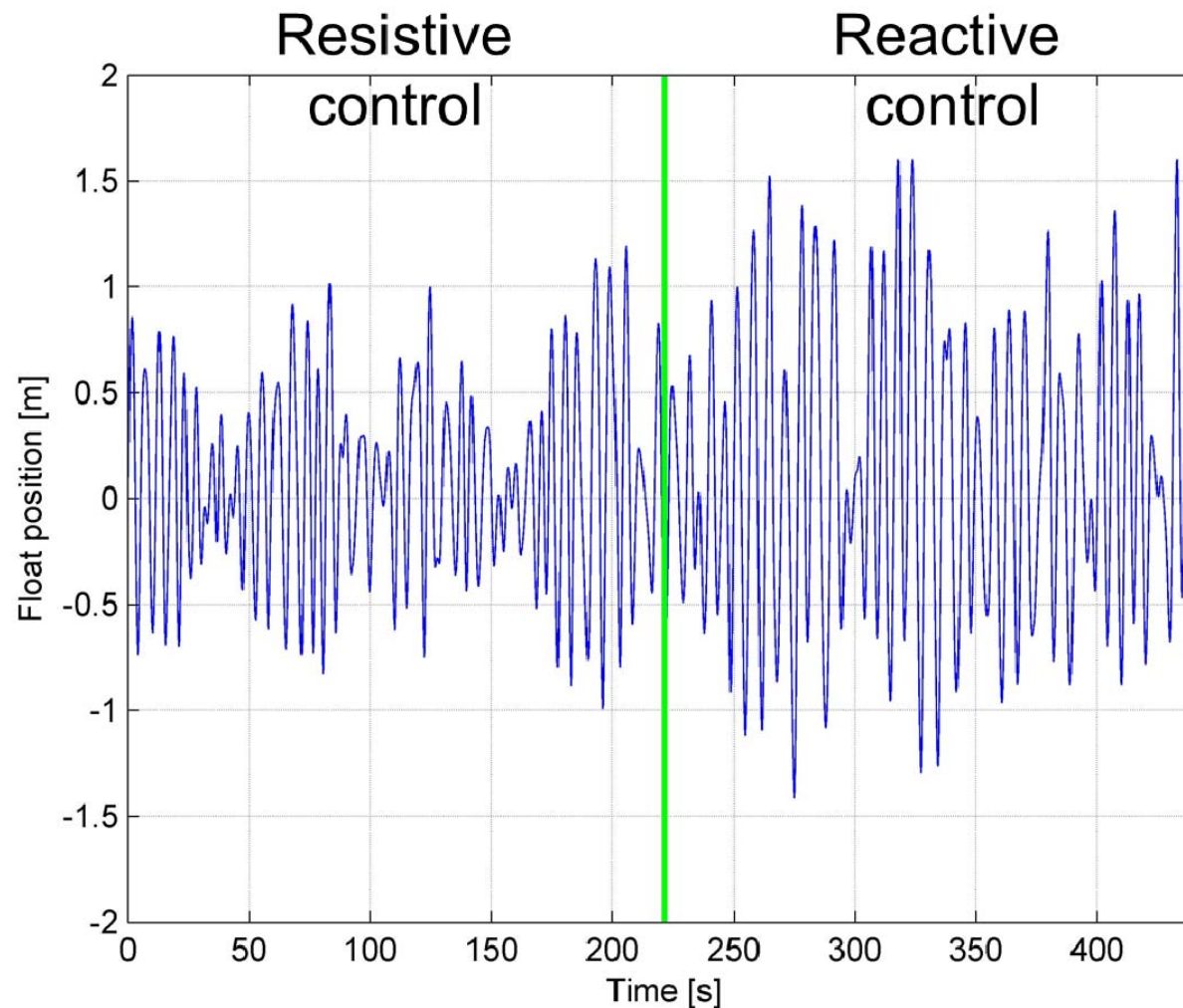
Forces depends strongly on the choise of control strategy



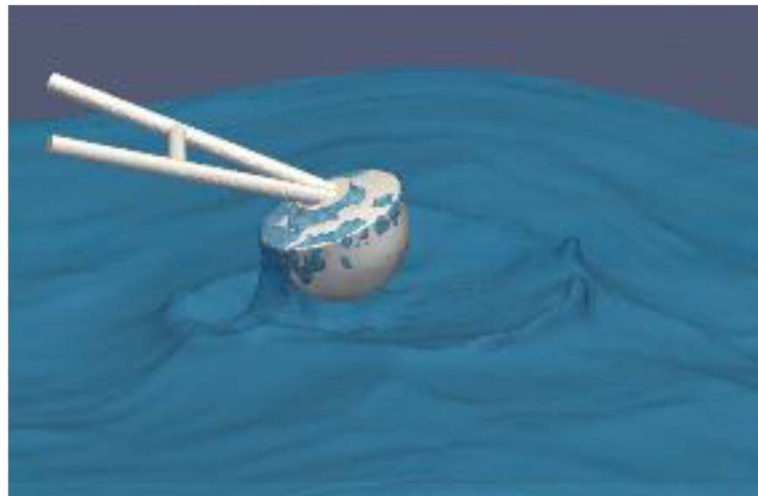
Constraints on eg. forces are more important when applying advanced control



Motions depends strongly on the choice of control strategy



How do we perform numerical calculations?



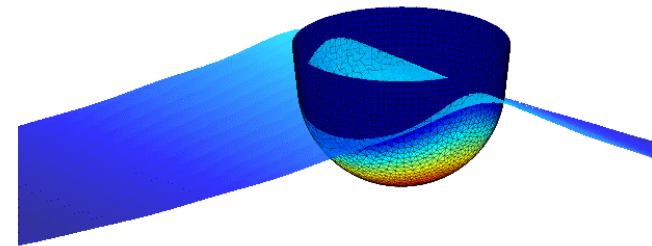
Total pressure variation on C5 float hull. Red colour is 3551 Pa

Water depth: 3.00 m
Wave height: 1.00 m
Wave period: 2.00 s
Wave direction: 40.00 °
Current speed: 0.00 m/s
Water level: -0.80 m

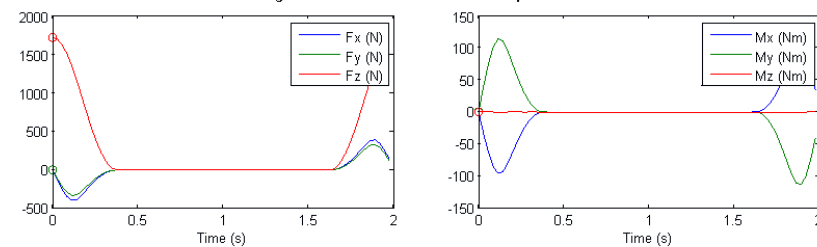
Float top cover is off

Nisum float shape
Nisum scale

Stream function wave theory
Fixed float
No structure interaction



Resulting forces and moments at centre of top cover



In the following we will only consider a linear potential wave model as this is still today the only practical method to perform optimization.

Description of harmonic oscillations using complex and real notation

Definition of a complex number:

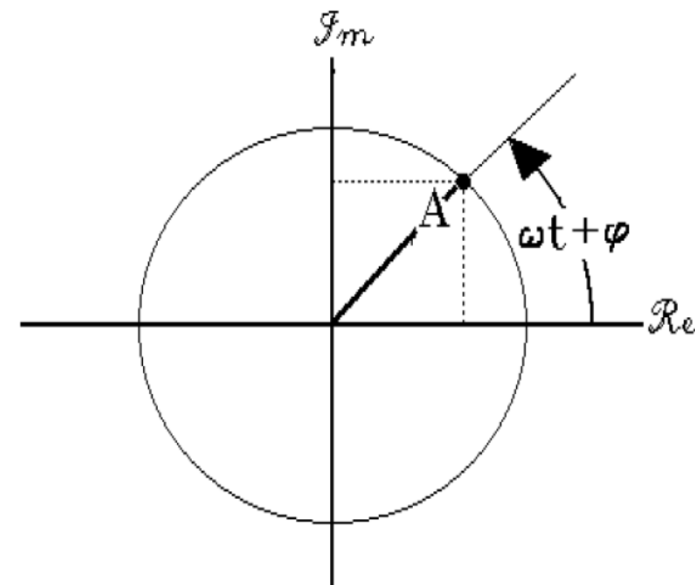
A complex number A is an ordered pair of real numbers $[a, b] \equiv [a + ib]$.

a is the real part of A , $a = \text{Re}(A)$,

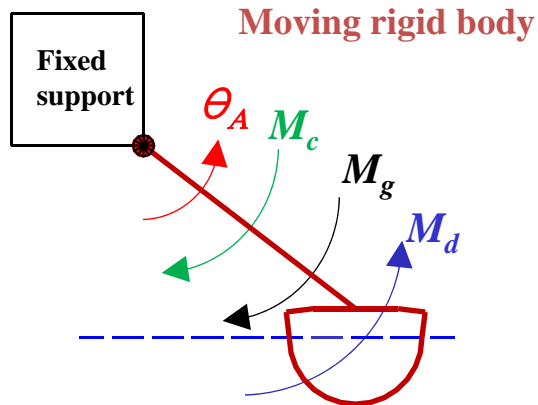
b is the imaginary part, $b = \text{Im}(A)$.

Complex and real representation of wave elevation:

$$\eta(t) = \text{Re}(A_w e^{i\omega t}) = |A_w| \cos(\omega t + \delta_w)$$



Equation of motion in frequency domain



Newton's second law:

$$J\ddot{\theta}_A = M_d - M_g - M_c \quad (1)$$

Equation 1 is expanded to:

$$J\ddot{\theta}_A = M_{hs} + M_r + M_{ex} - M_c \quad (2)$$

Hydrostatic moment:

$$M_{hs} = M_b - M_g = -k_h \cdot \theta_A$$

Radiation moment:

$$M_r = -m_h \cdot \ddot{\theta}_A - c_h \cdot \dot{\theta}_A \quad (3)$$

Wave excitation moment:

$$M_{ex} = \text{Re}(M_e \cdot e^{i\omega t})$$

Control moment:

$$M_c = m_c \cdot \ddot{\theta}_A + c_c \cdot \dot{\theta}_A + k_c \cdot \theta_A$$

where:

k_h : Hydrostatic stiffness coefficient

m_h : Hydrodynamic added mass coefficient

c_h : Hydrodynamic damping coefficient

M_e : Complex amplitude for wave excitation force, $M_e = H_{e\eta} \cdot A_w$

$H_{e\eta}$: Frequency response function for wave excitation moment (complex)

A_w : Wave amplitude (complex)

(4)

Equation of motion and solution in frequency domain

By inserting (3) into (2) and re-arranging the following is obtained:

$$M\ddot{\theta}_A + C\dot{\theta}_A + K\theta_A = \text{Re}(M_e e^{i\omega t}) = |M_e| \cos(\omega t + \delta_e) \quad (5)$$

where:

$$\begin{aligned} M &= J + m_h + m_c \\ C &= c_h + c_c \\ K &= k_h + k_c \\ M_e &= H_{e\eta} \cdot A_w \end{aligned} \quad (6)$$

The solution to (5) is a harmonic motion:

$$\begin{aligned} \theta_A(t) &= \text{Re}(A_A e^{i\omega t}) = |A_A| \cos(\omega t + \delta_A) \\ \dot{\theta}_A(t) &= \text{Re}(i\omega A_A e^{i\omega t}) \\ \ddot{\theta}_A(t) &= \text{Re}(-\omega^2 A_A e^{i\omega t}) \end{aligned} \quad (7)$$

where the motion amplitude is given by:

$$A_A = \frac{M_e}{K - \omega^2 M + i\omega C}, \quad |A_A| = \frac{|M_e|}{\sqrt{(K - \omega^2 M)^2 + \omega^2 C^2}} \quad (8)$$

Power absorption in regular waves

The instantaneous absorbed power is calculated by:

$$P_a(t) = M_c(t) \cdot \dot{\theta}_A(t) \quad (9)$$

And the average absorbed power:

$$\bar{P}_a = \frac{1}{T} \int_0^T P_a(t) dt = \frac{1}{2} \omega^2 c_c |A_A|^2 \quad (10)$$

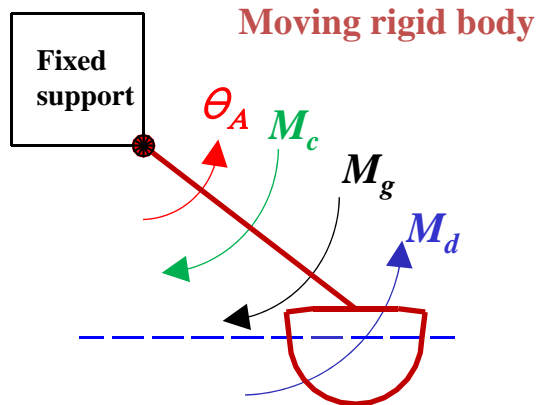
If using reactive control the optimum value of (10) may be found by choosing coefficients such that:

$$\begin{aligned} K - \omega^2 M &= 0 \\ c_c &= c_h \end{aligned} \quad (12)$$

In case of reactive control when (12) is fulfilled it can be shown that the optimum absorbed average power is:

$$\bar{P}_a = \frac{1}{8} \frac{|M_e|^2}{c_h} \quad (13)$$

Equation of motion in time domain



Newton's second law:

$$J\ddot{\theta}_A = M_d - M_g - M_c \quad (1)$$

Equation 1 is expanded to:

$$J\ddot{\theta}_A = M_{hs} + M_r + M_{ex} - M_c \quad (2)$$

Hydrostatic moment:

$$M_{hs} = M_b - M_g = -k_h \cdot \theta_A$$

Radiation moment:

$$M_r = -m_{h\infty}\ddot{\theta}_A - \int_{-\infty}^t h_r(t-\tau)\dot{\theta}_A(\tau)d\tau \quad (14)$$

Wave excitation moment:

$$M_{ex} = \int_{-\infty}^t h_{e\eta}(t-\tau)\eta(\tau)d\tau$$

Control moment:

$$M_c = m_c \cdot \ddot{\theta}_A + c_c \cdot \dot{\theta}_A + k_c \cdot \theta_A$$

where:

k_h : Hydrostatic stiffness coefficient

$m_{h\infty}$: Hydrodynamic added mass coefficient at infinite frequency

h_r : Impulse response function for wave radiation moment

$h_{e\eta}$: Impulse response function for wave excitation moment

A_w : Wave amplitude (complex)

Solving the equation of motion in time-domain

By inserting (14) into (2) and re-arranging the following is obtained:

$$(J + M_{h\infty})\ddot{\theta}_A(t) + \int_{-\infty}^t h_r(t-\tau)\dot{\theta}_A(\tau) d\tau + k_h\theta(t) + M_c(t) = M_e(t) \quad (15)$$

The convolution integral represents the memory effect in the radiation force. $M_{h\infty}$ is the limiting value of the added mass for $\omega \rightarrow \infty$. The coefficients may be calculated by:

$$h_r(\tau) = \frac{2}{\pi} \cdot \int_0^{\infty} C_h(\omega) \cdot \cos(\omega \cdot \tau) d\omega$$

$$M_{h\infty} = M_h(\omega) + \frac{1}{\omega} \int_0^{\infty} h_r(\tau) \cdot \sin(\omega \cdot \tau) d\tau \quad (16)$$

The equation is integrated numerically from initial conditions of position and velocity

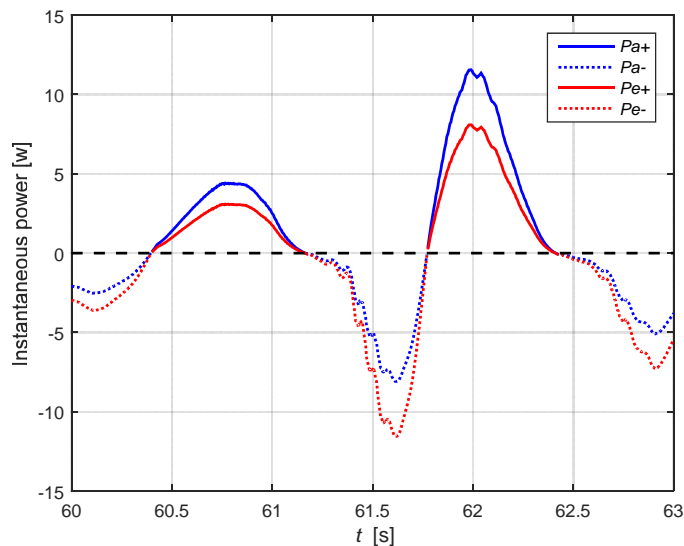
A simple Power Take Off efficiency model

Repetition: The instantaneous absorbed power is calculated by: $P_a(t) = M_c(t) \cdot \dot{\theta}_A(t)$

The generated electrical power is calculated from the harvested absorbed power by:

$$P_e = \eta_{PTO}^+ \cdot P_a^+(t) + \frac{1}{\eta_{PTO}^-} \cdot P_a^-(t) \quad (17)$$

P^+ is the positive power and P^- is the negative power. The efficiency is normally considered to be independent of the power direction, i.e. $\eta_{PTO}^+ = \eta_{PTO}^- = 0.7$ (in case of PTO efficiency of 70 %).



The effect of the formula on negative power flow may also be explained by the following two examples:

- If $\eta_{PTO}^- = 0.5$ and $P_a^- = -10kW$ then $P_e = \frac{1}{0.5} P_a^- = -20kW$. In this case the double amount of power is drawn from the grid (through the generator) compared to what we get out to the mechanical system.
- If $\eta_{PTO}^- \rightarrow 0 \Rightarrow$ It is necessary to draw infinitely large amounts of power from the grid (through the generator) to get any power out in the mechanical system.

Message:

- 1) Be aware of limitations in linear potential wave theory, and small amplitude approximations.
- 2) Applying advanced control methods in computer models may show very high increases in power production. However, in practice such methods are extremely difficult to apply in real irregular 3D waves. Wrong timing of latching/reactive controls is destructive for the buoy motion giving a lower power output.
- 3) Increase in power from Wave Energy Converters typically also increases forces and thereby costs. The optimal design is a balance between costs and production.

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